PS Algorithmen für verteilte Systeme

https://avs.cs.sbg.ac.at/

Exercise Sheet 0: Preliminaries

Exercise 1

Let A, B, C be sets. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Exercise 2

Let A, B, C, D be sets such that $C \subseteq A$ and $D \subseteq B$. Which of the following statements is true? Provide a proof or a counterexample.

(a) $(A \setminus C) \times (B \setminus D) \subseteq (A \times B) \setminus (C \times D)$

(b) $(A \setminus C) \times (B \setminus D) \supseteq (A \times B) \setminus (C \times D)$

Exercise 3

Let $r \neq 1$ and $n \in \mathbb{Z}_{\geq 0}$. Prove by induction that

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}.$$

Exercise 4

(a) Let $x \in \mathbb{R}$. Prove that $-|x| \leq x \leq |x|$.

(b) Let $a \geq 0$. Prove that $|x| \leq a$ if and only if $-a \leq x \leq a$.

(c) Use these two statements to proof the triangle inequality: for every $x, y \in \mathbb{R}$ we have $|x + y| \leq |x| + |y|$.

Exercise 5

Prove the following statements:

(a) $\log(n) = O\left(n^{o(1)}\right)$;

(b) $n^{\left(1+\frac{1}{14n}\right)^{7n}} = O\left(n^{\sqrt{e}}\right)$;

(c) Let $f\colon \mathbb{R} \longrightarrow [1, \infty]$ be a function such that $f(n) = \Omega(n)$. Then $1/f(n) = O(1/n)$.