PS Algorithmen für verteilte Systeme

https://avs.cs.sbg.ac.at/

Übungsblatt 1

Aufgabe 1

Deutsch: Zeigen Sie, dass, für passende Konfiguration der IDs, der Clockwise Algorithmus in einem synchronen, nicht-anonymen, Ring mit n Knoten an, bestenfalls $O(n)$ Nachrichten und schlechtestenfalls $\Omega(n^2)$ Nachrichten versendet.

English: Show that, for a certain configuration of IDs, the Clockwise algorithm in a synchronous, non-anonymous ring with n nodes, in the best case $O(n)$ messages and in the worst case $\Omega(n^2)$ messages are sent.

Aufgabe 2

English: Given is a synchronous, uniform ring with unique identifiers. For the clockwise algorithm, we assumed that each node has a designated clockwise and a designated counterclockwise port. Show that without this assumption, we can still appoint a leader in $O(n)$ rounds and $O(n^2)$ messages.

Aufgabe 3

English: Given a synchronous, uniform ring with unique identifiers on n nodes, denoted by $v_1, \ldots, v_n$. Let $\mathcal{A}$ be any deterministic algorithm that finds a unique leader. In this exercise, we will show lower bounds on the number of rounds $\mathcal{A}$ performs and the number of messages $\mathcal{A}$ sends.

(a) In this part, we show that $\mathcal{A}$ needs at least $n/4$ rounds. For simplicity, you may assume that $4|n$, i.e., $n/4$ is an integer.

Hints:

(i) Recall that for showing a lower bound, it is enough to give a particular instance on which the algorithm must take at least $n/4$ rounds.

(ii) Suppose $\mathcal{A}$ takes k rounds. Note that the ouput of v_i only depends on

$$v_{i-k}, v_{i-k+1}, \ldots, v_i, \ldots, v_{i+k-1}, v_{i+k}.$$

In particular this means that if $\mathcal{A}$ takes $n/4$ rounds, the output of each node can depend only on its half of the ring.

(iii) Suppose for contradiction that $\mathcal{A}$ always takes at most $n/4$ rounds. Create two cycles C_1, C_2 each of $n/2$ nodes, where $\mathcal{A}$ appoints leaders l_1 and l_2. Show that you can cut and paste these cycles such they form a cycle of length n, where $\mathcal{A}$ must appoint both l_1 and l_2 as leaders.

(b) Show that $\mathcal{A}$ sends at least $\Omega(n)$ messages.

Aufgabe 4

English: In the second lecture, you have seen the Radius Growth algorithm by Hirschberg and Sinclair. In this algorithm the radius grew in each iteration by a factor 2. Investigate what happens to the number of rounds and message complexity when we grow the radius by a factor $k \geq 2$ in each iteration. Prove your bounds on the number of rounds and message complexity.